12. Free Convection in Non-Newtonian Fluids from Heated Objects

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Abstract

This paper presents an overview of our research activity in the field of free convection in non-Newtonian fluids from variously shaped heated objects. In particular, consideration is given to two broad classes of fluids, namely, power-law fluids (shear-thinning and shear-thickening type) and Bingham plastic fluids. We have sought numerical solutions to the coupled momentum and energy equations within the framework of Boussinesq approximation to capture the temperature-dependence of the liquid density; all other thermo-physical properties are, however, assumed to be independent of temperature within the narrow range of temperature differences imposed in the system. The present results span wide ranges of Grashof number, Prandtl number and power- law index for a range of shapes including a sphere, a horizontal cylinder, elliptic cylinders of various cross-sections, a semi-circular cylinder and a square bar maintained at a constant temperature which is greater than that of the surrounding liquid. Extensive results on isotherm contours and streamline patterns and on Nusselt number are presented to delineate its scaling with Grashof number, Prandtl number and power-law index. Finally the present results are shown to be in good agreement with the scant experimental results available in this field. The paper is concluded by elucidating the role of shape and orientation of the heated object on free convection. The universal appeal of a composite parameter, akin to the Rayleigh number, in correlating the Nusselt number results for a wide variety of 2-D axisymmetric shapes is demonstrated. Finally, additional challenges posed by the Bingham plastic fluids are briefly discussed by way of free convection from a heated cylinder submerged in quiescent Bingham plastic fluids.

1. Introduction

Whenever there exists a temperature gradient in a fluid, heat transfer occurs by conduction and convection from the region of high temperature to that of low temperature. In the absence of a mechanical device to facilitate fluid motion, temperature-dependent density of the fluid gives rise to the buoyancy-induced flow which, in turn, transfers heat by the so-called free convection. Conversely, this contribution, howsoever small, is always present in most heat transfer applications. Of course, as the strength of the forced convection diminishes (indicated by vanishingly small values of the corresponding Reynolds number, Re), the contribution of free convection progressively increases. The relative importance of the free and forced convection mechanisms is quantified in terms of the familiar Richardson number, Ri, which is defined as, $Ri = Gr/Re^2$. Here, the Grashof number is a measure of the strength of the buoyancy-induced flow and the Reynolds number is that of the forced convection. Naturally, the pure forced convection limit is characterized by Ri = 0 (Gr = 0) whereas $Ri \rightarrow \infty$ corresponds to the pure free convection limit (Re = 0). Suffice it to add here that $Ri \sim O(1)$ corresponds to the conditions when the buoyancy-induced velocity is comparable to the imposed velocity. Thus, the contribution of the free convection to the overall heat transfer increases with the increasing value of the Richardson number. Notwithstanding the fact that free convection is always present, typical examples include heat losses from pipes carrying hot process streams like steam or water, storage tanks, high-temperature process equipment like distillation columns, reactors, etc., all of which are exposed to ambient conditions. In addition to such overwhelming pragmatic significance of free convection, the momentum and energy equations are coupled via the body force term and therefore the study of buoyancy-induced transport also constitutes an important sub-class of problems within the realm of transport phenomena. Consequently, over the years, much progress has been made in this field as far as the free convection transport in simple Newtonian fluids like air and water is concerned for most geometric configurations of practical interest. Excellent treatises are available on this subject (Martynenko and Kharmastov, 2005).

In contrast, it is readily conceded that most fluids of macromolecular (polymeric melts and solutions, protein solutions) and of multiphase nature (foams, emulsions, suspensions, for instance) encountered in a broad spectrum of industrial settings including polymer, food, pharmaceutical, personal and health care products, lubricants like grease, drilling muds, biological fluids do not conform to the simple Newtonian postulate. Instead, such "structured" fluids exhibit a range of rheological complexities including shear-thinning and shear-thickening viscosity, yield stress, visco-elasticity, thixotropy, etc. (Chhabra, 2006; Chhabra and Richardson, 2008). Naturally, it is not possible to consider all these aspects simultaneously and, in order to keep the level of complexity at a tractable level, it seems reasonable to begin with the simplest and possibly also the commonest type of non-Newtonian aspect, namely, shear-thinning and shear-thickening fluid behaviour which is usually approximated by the simple two-parameter power-law model which is written, in simple shear flow, as follows:

$$\tau = \mathrm{m}\left(\dot{\gamma}\right)^{\mathrm{n}} \quad \ldots\ldots\ldots\ldots (1)$$

In eq. (1), τ is the shear stress produced in the fluid when it is sheared at the rate of shear, $\dot{\gamma}$. The pre-factor, m, is known as the consistency index and it is a measure of the fluid consistency. Conversely, it can be viewed as the value of the fluid viscosity at a shear rate of $\dot{\gamma} = 1\,\mathrm{s}^{-1}$. The index, n, is known as the power-law index. Evidently, $\mathrm{n} < 1$ indicates shear-thinning behaviour whereas $\mathrm{n} > 1$ corresponds to the so-called shear-thickening behaviour. Of course, n = 1 denotes the standard Newtonian fluid behaviour. From a practical standpoint, many polymeric fluids and suspensions exhibit values of the power-law index in the range $\sim 0.2 \leq \mathrm{n} \leq 0.6$. On the other hand, thick pastes and suspensions (corn flour in water, starch in water, for instance) exhibit values of $\mathrm{n} > 1$ thereby leading to shear-thickening behaviour. So the major thrust of this work is on studying the laminar free convection from variously shaped heated objects in quiescent power-law media.

Next, we provide a brief introduction to the additional challenges posed by the so-called yield-stress fluids which exhibit elastic solid-like behavior below a threshold stress level (yield stress) and hence the flow domain is spanned by fluid-like and solid-like zones (Bird et al., 1983; Barnes, 1999). The simplest viscosity model to capture the yield-stress behaviour is the so-called Bingham plastic model which, in simple shear, can be written as:

$$\tau = \tau_0 + \mu_{\mathrm{B}}\left(\dot{\gamma}\right) \text{ for } |\tau| > |\tau_0| \quad \ldots\ldots\text{ (2a)}$$

$$\dot{\gamma} = 0 \qquad \text{for } |\tau| < |\tau_0| \quad \ldots\ldots\text{(2b)}$$

In eq. 2(a), τ_0 is the so-called yield stress and μ_B is the Bingham plastic viscosity. Whether the true yield stress exists or not has been a matter of debate (Barnes, 1999), the flow behaviour of many practical materials is conveniently approximated by eq. 2, Intuitively, it appears, with these fluids, convection will dominate the fluid-like regions whereas the unyielded (solid-like) regions permit heat transfer only by conduction thereby lowering the overall rate of heat transfer. In this talk, we present an overview of our recent work in this field which has been reviewed extensively elsewhere (Chhabra, 2011)

2. Analysis and Dimensional Considerations

Undoubtedly, the major thrust of research in this field is on the prediction of the heat transfer coefficient in a given application where free convection is the sole mechanism of heat transfer. From a theoretical standpoint, the momentum and energy equations are coupled via the buoyancy term and therefore, these need to be solved simultaneously. This aspect precludes the possibility of general rigorous solutions even for Newtonian fluids (Martynenko and Kharmastov, 2005). Therefore, early attempts at such analysis of free convection are based on the solution of the boundary layer equations which implicitly assume infinitely large values of the Grashof number and/or Prandtl number so that the thin boundary layer assumption can be justified as well as the curvature effects can be neglected. Admittedly this approach has led to reliable scaling of the skin friction and Nusselt number with Grashof and/or Prandtl number, it does not capture the wake region. Nor does it help delineate the values of the Grashof number and/or of the Prandtl number beyond which this analysis is applicable. Notwithstanding these limitations, this approach has been widely used for axisymmetric shapes like sphere, cylinder, spheroids, and of course, plane surface, etc. The other limiting case of the vanishingly small values of the Grashof number is treated via the asymptotic expansion technique such as that used by Singh and Hasan (1983) for a sphere and Nakai and Okazaki (1975) for a cylinder. Therefore, neither of these approaches is valid at finite values of the governing parameters (Grashof number and/or Prandtl number) and, more importantly, these can be only employed for axisymmetric shapes which are free from geometric singularities like a square or a triangular prism. Indeed the results for Newtonian fluids based on the numerical solution of the complete momentum and energy equations even for regular shapes like a plate or a cylinder or a sphere have been reported during the past 25-30 years only (Martynenko and Kharmastov, 2005). Suffice it to say here that based on a combination of the analytical, numerical and experimental studies, reliable methods are now available for the estimation of skin friction and Nusselt number in the free convection regime over most ranges of practical interest in Newtonian fluids, at least for the regular shapes of spheres, cylinders or plates.

For a given geometric configuration, most of these results are expressed by the following generic form:

$$\mathrm{Nu} = \mathrm{f}(\mathrm{Gr}, \mathrm{Pr}) \qquad \text{……..} (3)$$

Where the general definition of Nusselt number, Nu = hd/k. Here h is the convective heat transfer coefficient, d is a characteristics linear dimension like diameter or radius for sphere and cylinder and k is the thermal conductivity of the fluid. Similarly, the Grashof number (Gr) and Prandtl number for a Newtonian medium are defined as:

$$\mathrm{Gr} = \frac{\rho^2 \mathrm{d}^3 \left(\mathrm{g} \beta \Delta \mathrm{T} \right)}{\mu^2} \qquad \text{……..} (4)$$

$$\mathrm{Pr} = \frac{\mathrm{C_p} \mu}{\mathrm{k}} \qquad \text{……..} (5)$$

where ρ is the fluid density; g is acceleration due to gravity; β is the coefficient of expansion and ΔT is the temperature difference between the ambient fluid and the heated object; μ is the Newtonian viscosity; C_p is the thermal heat capacity.

The actual functional relationship embodied in eq. (3) depends upon many other aspects including the nature of the boundary condition prescribed on the surface of the heated object (constant temperature or constant heat flux), laminar or turbulent flow conditions, viscous dissipation, temperature-dependence of the physical properties, etc. Extensive compilations encompassing wide ranging shapes and conditions are available in the literature (Martynenko and Kharmastov, 2005).

The analogous literature for power-law fluids is neither as extensive nor coherent as that for Newtonian fluids (Shenoy and Mashelkar, 1982; Chhabra, 2006). Early pioneering effort in this field is due to Acrivos (1960) who presented limited results for the laminar free convection in power-law fluids from axisymmetric shapes including the case of a sphere, a cylinder and a plate. Subsequently, this work has been confirmed by the other studies (Stewart, 1971; Meissner et al., 1994). However, as noted previously, such an analysis neither accounts for the wake region nor is applicable at finite values of the Grashof and Prandtl numbers. Therefore, one must resort to the numerical solutions of the complete momentum and energy equations to circumvent these limitations.

During the past five years or so, reliable numerical predictions of the detailed kinematics of flow (streamline and isotherm contours), distribution of Nusselt number along the surface of heated objects and the overall mean Nusselt number have been reported for power-law fluids. Prahashanna and Chhabra (2010, 2011) studied laminar natural convection from an isothermal sphere and cylinder over wide ranges of power-law index (n) and Grashof and Prandtl numbers. Subsequently, analogous results have been reported for elliptical cylinders (Sasmal and Chhabra, 2012b), square and rotated square cylinders (Sasmal and Chhabra, 2011, 2012a) and semi-circular cylinders in different configurations (Chandra and Chhabra, 2012; Tiwari and Chhabra, 2013). This paper provides an overview of our work in this field.

Dimensional considerations as applied to the appropriate momentum and energy equations together with the relevant boundary conditions lead to the following definitions of the Grashof number and present emerge power-law fluids

$$\mathrm{Gr_p} = \frac{\rho^2 (\mathrm{d})^{\mathrm{n}+2} (\mathrm{g}\beta\Delta\mathrm{T})^{2-\mathrm{n}}}{\mathrm{m}^2} \quad \text{……….} (6)$$

$$\mathrm{Pr_p} = \frac{\rho \mathrm{C_p}}{\mathrm{k}} \left(\frac{\mathrm{m}}{\rho}\right)^{\left(\frac{2}{1+\mathrm{n}}\right)} (\mathrm{d})^{\left(\frac{1-\mathrm{n}}{1+\mathrm{n}}\right)} \left(\mathrm{dg}\beta\Delta\mathrm{T}\right)^{\frac{3(\mathrm{n}-1)}{2(\mathrm{n}+1)}}$$

………(7)

Note that much more unwieldy forms of the Grashof and Prandtl numbers for power-law fluids, albeit these do reduce to their limiting forms, as given in eq. (4) and eq. (5) for n = 1. In addition, the power-law index, n, is a dimensionless parameter in its own right. Thus, for a power-law fluid, the functional relationship denoted by eq. (3) is reformulated as follows:

$$\mathrm{Nu} = \mathrm{f_1}(\mathrm{Gr_p}, \mathrm{Pr_p}, \mathrm{n}) \quad \text{……….}(8)$$

Both the experimental and numerical approaches have been used to establish the functional relationship implicit in eq. (8) using Gr_p, Pr_p and n. However, our recent experience suggests that the following composite parameter, Ω, is rather more effective in consolidating the results for a range of geometric shapes studied thus far than the Grashof number and Prandtl number. It is defined as:

$$\Omega = Gr_p^{\frac{1}{2(n+1)}} Pr_p^{\frac{n}{3n+1}} \qquad \text{……….. (9)}$$

For n = 1 (Newtonian fluids), the composite parameter, Ω, is identical to the $Ra^{1/4}$ where the so-called Rayleigh number, Ra, is defined as $Ra = Gr \cdot Pr$. The available experimental and numerical results for laminar free convection in Newtonian fluids conform to $Nu \propto Ra^{1/4}$, i.e., $Nu \propto \Omega$. By analogy, one can thus re-cast the relationship of eq. (8) as follows:

$$Nu = a\Omega^b \qquad \text{………(10)}$$

Naturally, eq. (10) is applicable for a fixed geometric configuration. Indeed, Table 1 presents a summary of the currently available results on laminar free convection in power-law fluids thereby showing the universal appeal of the composite parameter.

Broadly speaking, all else being equal, shear-thinning fluid behaviour (n < 1) promotes heat transfer over and above that seen in Newtonian media. Of course, shear-thickening behaviour impedes it. Indeed, it is possible to realize up to 100% augmentation in Nusselt number in shear-thinning fluid under appropriate conditions. Also, the fact that $b \approx 1$ in almost all cases summarized in Table 1 demonstrates the universal appeal of the composite parameter, Ω, at least in conformity with the scaling suggested by the boundary layer considerations. The effect of geometry reflected by the value of a is seen to be significant. Before closing this sub-section, it is important to reiterate here the assumptions inherent in the numerical studies which form the basis of the results reported in Table 1. Firstly, these results are based on the assumption of constant physical properties and negligible viscous dissipation effects thereby limiting their validity to the situations wherein ΔT is not excessive and one can thus evaluate k, C_p, m, n etc. at the mean film temperature. Secondly, only the results corresponding to the constant wall temperature conditions are included in Table 1. Lastly, the flow is assumed to be laminar in all cases.

Table 1: Values of a and b in Eq. (10) for different shapes.

Reference	Shape	Linear dimension	a	b
Prhashanna & Chhabra (2010)	Sphere	R	2.00	0.72
Prhashanna & Chhabra (2011)	Horizontal circular cylinder	d	1.19	0.89
Sasmal & Chhabra (2012b)	Elliptic cylinder ($0.2 \le E \le 5$)	2a	0.83	0.89
Chandra and Chhabra (2012)	Semi-circular cylinder (flat base upward)	d	0.93	0.79
Tiwari and Chhabra, (2013)	Semi-circular cylinder (flat base downward)	d	0.72	0.90
Sasmal & Chhabra (2011)	Square cylinder	B	0.60	0.92
Sasmal & Chhabra (2012a)	Tilted square cylinder ($\alpha = 45^\circ$)	B	0.76	0.92

Next, we turn our attention briefly to the case of free convection in Bingham plastic fluids where the fluid behaviour is characterized in terms of a yield stress (τ_0) and plastic viscosity (μ_B), as suggested by eq. 2. Since once the prevailing stress level exceeds the yield stress, τ_0, such a material behaves like a Newtonian fluid with viscosity μ_B, one can use the same definitions of the Grashof number and Prandtl number, as that given by eq. (4) and eq. (5) for Newtonian fluids, or one can work in terms of the Rayleigh number, $Ra = Gr \cdot Pr$. However, an additional dimensionless group, namely a Bingham number emerges in this case. For free convection, it is defined as:

$$Bn = \frac{\tau_0}{\mu_B}\sqrt{\frac{d}{g\beta\Delta T}} \quad \text{…………..(11)}$$

Intuitively, it appears that with the increasing value of the yield stress, i.e., the Bingham number, the flow domain is increasingly dotted by the unyielded (solid-like) regions where heat transfer occurs only by conduction. This point is illustrated by showing some results from a heated circular cylinder which is submerged in a body of Bingham plastic fluid (at a lower temperature than the cylinder) filled in a square duct whose walls are also at the same temperature as the fluid (Fig. 1). These results have been recently reported by Sairamu et al. (2013). Figure 2 shows typical results on the so-called yielded and unyielded (shaded) regions. Evidently, there are regions which act like a solid material and this has an adverse influence on the overall heat transfer. Fig. 3 shows that beyond a limiting value of the Bingham number (Bn_{max}), the Nusselt number is constant which is identical to the corresponding conduction limit (Sairamu et al., 2013).

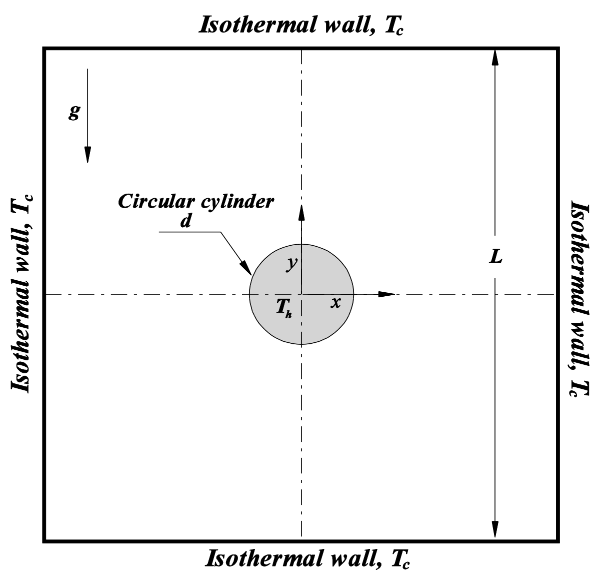

Figure 1: Schematics of the flow configuration

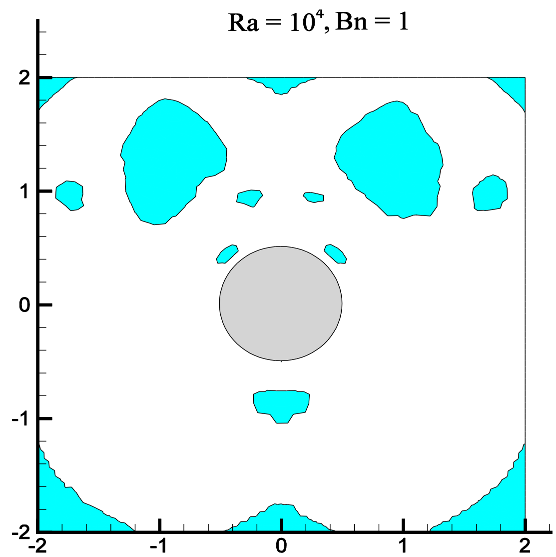

Figure 2: Typical structure of yield/un-yielded regions

In this case, it is naturally advantageous to correlate the Nusselt number results by two expressions. Thus, for instance, Sairamu et al. (2013) reported the following equations:

$$Nu = 2.585 \qquad \text{for } Bn \geq Bn_{max} \quad \ldots.(12a)$$

$$Nu = 2.585 + 0.0095 Ra^{1/4} (Bn_{max} - Bn)^{2.3} \quad \ldots\ldots\ldots\ldots(12b)$$

Thus, heat transfer in such fluids not only tends to be inherently poor, but its prediction also necessitates knowledge about the value of Bn_{max} a priori. Indeed, Bn_{max} shows an intricate dependence not only on the geometrical configuration but also on the type of boundary conditions as well as on the range of Rayleigh numbers of interest.

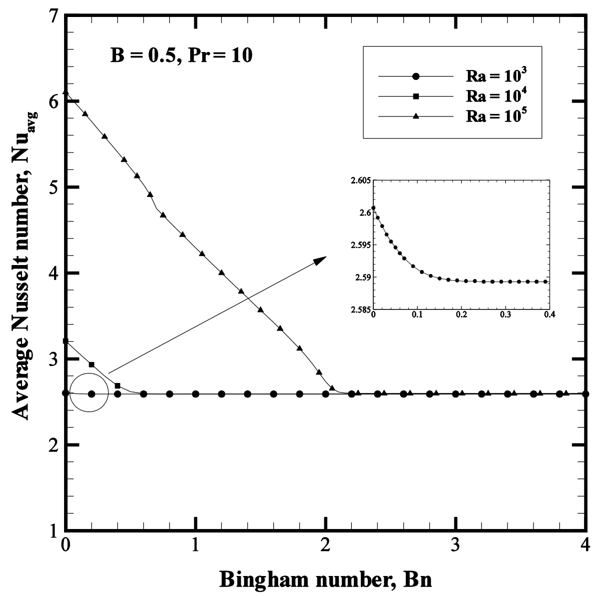

Figure 3: Dependence of average Nusselt number with Bingham number and Rayleigh number.

3. Concluding Remarks

In this work, an attempt has been made to provide an overview of the nature of buoyancy-induced free convection in two types of non-Newtonian fluids, namely, power-law fluids and Bingham plastic fluids. Reliable results available in this field are not only rather scant but these are also of very recent vintage. Broadly, shear-thinning and shear-thickening fluids behave in a qualitatively similar fashion as Newtonian fluids. However, shear-thinning behaviour can enhance the rate of heat transfer by up to 100% under appropriate conditions whereas the shear-thickening fluid behaviour somewhat impedes the rate of heat transfer. Analogous buoyancy-induced flow in yield stress fluids has been studied even less extensively and the field is still in its infancy. In this case also, there are parts of region which are dominated by conduction thereby lowering the overall rate of heat transfer. This rugged terrain of non-Newtonian transport phenomena deserves more attention that it has received thus far.

References

1. Acrivos, A., A theoretical analysis of laminar natural convection heat transfer to non-Newtonian fluids, AIChE J. 6 (1960) 584-590.
2. Barnes, H.A., Yield stress—a review or 'πανταρ ε ι'—everything flows?, J. Non-Newtonian Fluid Mech. 81 (1999) 133-178.
3. Bird, R. B., Dai, G.C., Yarusso, B. J., The rheology and flow of viscoplastic materials, Rev. Chem. Eng. 1 (1983) 1–70.
4. Chandra, A., Chhabra, R.P., Laminar free convection from a horizontal semi-circular cylinder to power-law fluids, Int. J. Heat Mass Transfer 55 (2012) 2934-2944.
5. Chhabra, R. P., Bubbles, drops and particles in non-Newtonian fluids, 2nd edition CRC Press, Boca Raton, 2006.
6. Chhabra, R.P., Fluid flow and heat transfer from circular and non-circular cylinders submerged in non-Newtonian liquids, Adv. Heat Transfer, 100 (2011) 289-417.
7. Chhabra, R.P., Richardson, J.F., Non-Newtonian flow and applied rheology, 2nd edition, Butterworth-Heinemann, Oxford, 2008.
8. Martynenko, O.G., Khramtsov, P.P., Free convection heat transfer with many photographs of flows and heat exchange, Springer, Berlin, 2005.
9. Meissner, D.L., Jeng, D. R., DeWitt, K.J., Mixed convection to power-law fluids from two-dimensional or axisymmetric bodies, Int. J. Heat Mass Transfer 37 (1994) 1475-1485.
10. Nakai, S., Okazaki, T., Heat transfer from a horizontal cylinder wire at small Reynolds and Grashof number-I, Int. J. Heat Mass Transfer 18 (1975) 387-396.
11. Prhashanna, A., Chhabra, R.P, Free convection in power-law fluids from a heated sphere, Chem. Eng. Sci. 65 (2010) 6190–6205.
12. Prhashanna, A., Chhabra, R.P, Laminar natural convection from a horizontal cylinder in power-law fluids, Ind. Eng. Chem. Res. 50 (2011) 2424–2440.
13. Sairamu, M., Nirmalkar, N., Chhabra, R.P., Natural convection from a circular cylinder in a confined Bingham plastic fluids, Int. J. Heat Mass Transfer 60 (2013) 567-581.
14. Sasmal, C., Chhabra, R.P, Laminar natural convection from a heated square cylinder immersed in power-law liquids, J. Non-Newt. Fluid Mech. 166 (2011) 811–830.

15. Sasmal, C., Chhabra, R.P, Effect of orientation on laminar natural convection from a heated square cylinder in power-law fluids, Int. J. Therm. Sci. 57 (2012a) 121–125.
16. Sasmal, C., Chhabra, R.P, Effect of aspect ratio on natural convection in power-law liquids from a heated horizontal elliptic cylinder, Int. J. Heat Mass Transfer 55 (2012b) 4886–4899.
17. Shenoy, A.V., Mashelkar, R.A., Thermal convection in non-Newtonian fluids, Adv. Heat Transfer 15 (1982) 143-225.
18. Singh, S. N., Hasan, M.M., Free convection about a sphere at small Grashof number, Int. J. Heat Mass Transfer 25 (1983) 781-783.
19. Stewart, W.E., Asymptotic calculation of free convection in laminar three dimensional systems, Int. J. Heat Mass Transfer 14 (1971) 1013-1031.
20. Tiwari, A.K., Chhabra, R.P., Laminar natural convection in power-law liquids from a heated smi-circular cylinder with its flat side oriented downward, Int. J. Heat Mass Transfer 58 (2013) 553-567.