Mathematics and Applied Sciences

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MATHEMATICS is included as a subject of study in courses leading to the award of degrees both in arts and sciences in many Universities. What is its position among the branches of knowledge today ? It is true, in common with the arts subjects mathematics is built up by the creative imagination of man. An elegant piece of poetry, an interesting novel, a brilliant drama, a thoughtful essay, a well rendered piece of music or an arresting painting are all creations of imagination. They all end in stirring the human feelings and do not create anything new. But mathematics emanating from the same creative imagination builds up something concrete in the abstract. The zero, negative numbers, rationals, irrationals, matrices, quaternions are all there ; one who opens his eyes can see them and play about with them.

On the other hand mathematics differs from the experimental sciences also. The methods of the two, in spite of many common points, show a distinct diversity. Mathematics adopts only the faultless deductive process of reasoning. Whether it is the indirect method of proof or that process with the resounding misnomer—mathematical induction—the course is precise logic and deduction. A statement might have been verified in as many cases as it has been tried. It is still a hypothesis or at best a probable truth until it is proved in the general.

Contrasted with this the experimental sciences use the inductive method largely in addition to the deductive method in a few cases. They are full of theories, improved laws and hypotheses. The procedure is from the special to the general enunciating laws which are not inescapable conclusions. The vast knowledge built up by the help of thermodynamics hinges on two facts based on human experience. However, this method is not without benefits ; the experimental sciences have made large strides using this method. Basing on a set of experiments generalizations are made. Frequent failures are encountered immediately on application of these generalisations and thought is stimulated to understand the secrets of nature. This process is inevitable. It is in the nature of things encountered. It is not possible to adopt always only the strict deductive method. The human urge for knowledge drives us to the inductive process.

Mathematics thus stands on a unique pedestal. It is at a higher level compared with the arts subjects, the social sciences, the experimental sciences or applied sciences. *It is the cream of knowledge and the queen of sciences.* For the applied scientist it is the most powerful tool to expand and create.

The origin of mathematics is closely associated with the practical needs of daily life. Even in the primitive stages of society the number property was realized and associated with the everyday business of man. Geometry materialized from the necessity of repeated land measurements. Even after considerable development mathematics was wedded to natural phenomenon. It was looked upon solely as an instrument for the study of nature.

When it dawned that Euclid's assumptions were not the only sacred truths which should be held as eternal and man found himself free to choose his own speculative assumptions as the starting-point, it marked an achievement in the history of mathematics,

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history of thought and history of civilization. Mathematics emerged out of the nest to occupy a greater role. Freed once for all from the shackles mathematicians engaged themselves in pursuit of fields of knowledge which were not called upon by natural phenomenon to be pursued. They had the thrill of creating new things whether their creations were used for any purpose or not by man. Enquiry became more and more specialized. Attempts were made to unify diverse fields of investigation. The trend was towards the extraction of the essential general feature and the principle of abstraction emerged. "The colourful individual was left behind to embrace the *sublime abstract general.*"

At a very early stage it was realized by the mathematicians that the real value of knowledge lay in the true knowledge itself and not in any test of its application to the needs of the human being. Existence of applications, sensational or otherwise, was not the criterion to assess the value of a mathematical result. Many would demand as proof of its value utilitarian application of mathematics to our daily lives. Others react in disgust to this demand for practical applications so violently that a famous Cambridge Don is said to have proposed a toast "To pure mathematics and may it never be of any use to anyone."

The assiduous pursuit of knowledge for its own sake built up a large pile and vast storehouse of subject-matters readily available for any one if he could find a use for it. Many a scientist has found in this storehouse the necessary tools ready ground for him to solve his problems. The theory of conic sections, which had been studied for centuries as pure mathematics gave Newton, who observed the path of heavenly bodies to be approximately conic sections as was already known to Kepler, the clue which led to the inverse square law of gravitation. Dirac and Heisenberg found the Matrices awaiting them. Non-euclidean geometry made Einstein emerge to base his theory of the structure of the universe on it. According to him the geometry of the physical world was not euclidean but it was Reimannian replacing the gravitational force by a geometrical law. The imaginary "i" permeated the study of the problems of electrical circuits and today it controls the flow of electricity through the conductors much more closely than the sluices of Mettur dam can ever hope to control the flow of the waters of the river Cauveri. The natural phenomenon of Brownian movement, discovered by and named after the great botanist Robert Brown, giving a visible demonstration of the movements of molecules of a liquid, has been found to be naturally set up by means of a freak disclosed by the purely mathematical study of the Fourier theorem—namely a continuous function which has no derivative at any point.

Returning to the standard of values insisting on practical utility one cannot fail to be impressed by one of the most modern achievements resulting from the spectacular application of a mathematical conclusion arrived at in the sole pursuit of knowledge for its own sake—namely the atom bomb. It is very unfortunate that this should have been used first for the destruction of mankind and civilization. It is a piece of consolation, poor even though it is, that honest attempts are being made to harness atomic energy for human welfare and good of the world.

Reimannian geometry provided a clue to Einstein, one consequence of whose special theory of relativity was that energy must have both mass and weight. If a clock spring is wound it differs from the unwound spring in having added energy and must be heavier than the unwound spring. The increase in weight is too small for measurement with the existing most sensitive balances, nevertheless the *extra* weight can be calculated *exactly.* Thus matter and energy are not only interrelated but they are also interconvertible. By calculations, in which this mass-energy relationship which follows from Einstein's theory is applied, it can be shown that when four hydrogen atoms combine to give one helium atom, a fraction, 1/140, of the total mass of Hydrogen is converted into energy. A one ton projectile, if it contains one ounce of water and a ninth of this which is hydrogen is converted into helium, the energy derived by conversion of matter will be sufficient to shoot the projectile with a velocity of forty-four miles per second and raise it to a height of one thousand miles above the earth's surface. Four ounces of water will be sufficient to remove the ton weight completely out of the earth's field of attraction to start a trip to the moon.

Such a process of building heavier atoms from lighter ones on the low atomic weight end of the "Periodic table" is supposed to be taking place in the sun and is responsible for the enormous amount of energy given out by the planet. Building up of atoms on the high atomic weight end of the "Periodic table by man has been an accomplished fact and attempts to build up other atoms starting from hydrogen have been successful. These phenomenal applications of what were purely brilliant creations of the mathematical mind in the abstract will appeal even to the most "*Utilitarian and practical result*" minded of men.

Although a part of nature, man always apes and tries to improve on nature. He used a tree that had fallen across a stream by chance for crossing over and now he builds huge bridges to cross over mighty rivers. Looking at a docile slow moving animal he got by instinct the idea of perching on its back and transporting himself without effort. The natural craze for speed made him first domesticate the horse. To utilize fully the animal power he devised the cart so that more people in greater comfort could be transported than could be possible by sitting on its back. First he drifted over water on a floating log. He has now replaced these primitive methods by the automobile, the railways and ships. Imitating the birds he has evolved the heavier-than-air aircraft to shrink the world of long distances into a matter of hours. Enjoying nature's shelter in a cave in the beginning man has constructed beautiful buildings to live comfortably. Not satisfied with clothing himself using animal and vegetable fibres he has now created artificial fibres like rayon and nylon having tailored-to-taste qualities.

All these achievements require an intimate knowledge of materials, their stresses and strains, the forces which play on them, of their practicability of supporting the loads they are called upon to carry and the degree of safety. Man requires an accurate knowledge of heat conductance, of electrical transmission, and of chemical reactions. In the nature of things it is inherent that these ideas should be handled quantitatively and they should be expressed mathematically. Soon it was realised that precise, additional and underlying information could be obtained ultimately only by mathematical means.

So man turned to mathematics to search for the tools and after an initial success he discovered the vast storehouse of subject-matter that the mathematician had built up by his study, in the abstract, of subjects not connected with any natural phenomenon. Starting with the simple tools the more advanced techniques of mathematics were adopted. Half a century ago many an 'applied scientist' regarded differential and integral calculus to be a mystery beyond reach of many. Today the average student has to take these in the regular course of his studies in applied sciences. The solution of problems in an elegant way using compound numbers requires at least a superficial study of the elements of the complex variable. Solution of problems of various kinds in acoustical, electrical, aeronautical and chemical engineering has ushered in the partial differential equations, matrix algebra, operational calculus and other advanced techniques into the necessary minimum mathematical equipment of any applied scientist of today.

The fundamental method of approach consists in reduction of the physical phenomenon involved into a mathematical system and solving it. A close survey of the problem to be solved is carried out. A functional relationship is arrived at and information is extracted by tackling the rate or root problem, the extreme value problem, the area problem or the common value problem. The most difficult part of the process is the setting up of the mathematical system. It demands originality and inventive ability of thought some times bordering on intuition. In many cases the fundamental analysis is so highly complicated that recourse has to be taken to that powerful tool of dimensional analysis to obtain a relationship between the variables and determine the constants and form of the function by experimental procedure. In a number of problems various simplifying assumptions have to be made to eliminate a number of 'Uncertains.' To cover up this, after precise calculations, the final result is made up to five hundred or six hundred per cent its value. If so, one may wonder whether the close mathematical analysis and precise computation is worthwhile. It is and that alone gives us an idea of what it is that has got to be increased by five hundred per cent; whether six, sixty or six hundred.

The ultimate aim of all this mathematical operation is to arrive at something concrete or in other words to compute a numerical value. In this connection it is but apt to point out the indebtedness of the engineer or scientist to the mathematicians for the epoch-making discovery by Napier—the logarithms —by using which many involved computations are considerably simplified and some of the impossible ones are at least made feasible. While making actual computations to arrive at working figures the applied scientist may forget the spirit of mathematics and adopt ways repugnant to the mathematician. For the latter π is π but for the applied scientist it is 22/7 or at best 3.1416.

With all achievements using mathematical tools the applied scientist should not forget that a perfect structure is erected in nature by instinct. The spider weaves its web, a very complicated structure, without knowing what mathematics is. Instinct triumphs over calculation.