Fluid Flow and Heat Transfer in Helical Coils—A Review

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BEFORE the advent of shell-tube type heat exchangers, the coil type of heat exchangers were widely used. Even today these types of heat exchangers continue to be used in some of the industries such a glycerine evaporation and sugar juice concentration. In unit processes like sulphonation and nitration, a combined reaction cum-heat transfer unit necessarily employs coils for heat exchange.

Apart from the advantages of expediency and low cost of installation, helical coils offer enhanced heat transfer rates as compared with straight pipes under identical operating conditions.

This enhanced heat transfer depends, in addition to the usual factors such as fluid properties and velocity, on the geometrical design of the coil, which includes the following factors:—

(1) Ratio of the pipe diameter to the diameter of the helix of the coil.
(2) Centre to centre spacing between helices.
(3) Number of turns in the coil.

Many attempts have been made to correlate the heat transfer and fluid friction data in terms of the d/D ratio and the usual dimentionless groups. The purpose of this article is to enumerate the previous attempts in this direction.

Although coils were used since long in heat transfer apparatus, the conditions of fluid flow in coils were not investigated till 1927. In this Year Dean showed mathematically that increase in resistance to fluid flow in a coil over that in a straight pipe is a function of Reynold's number and the square root of the curvature ratio D/d which is termed Dean's criterion. Dean[1] assumed a stream line motion with the fluid at the centre flowing with twice the mean velocity. If the fluid is constrained to move in the curved pipe, the centrifugal action of the liquid at the centre gives it a tendency to move towards the outside wall of the pipe. On nearing the wall it divides into two streams which take opposite paths and give rise to a double circulatory effect which has been termed double helical flow (see Fig. A).

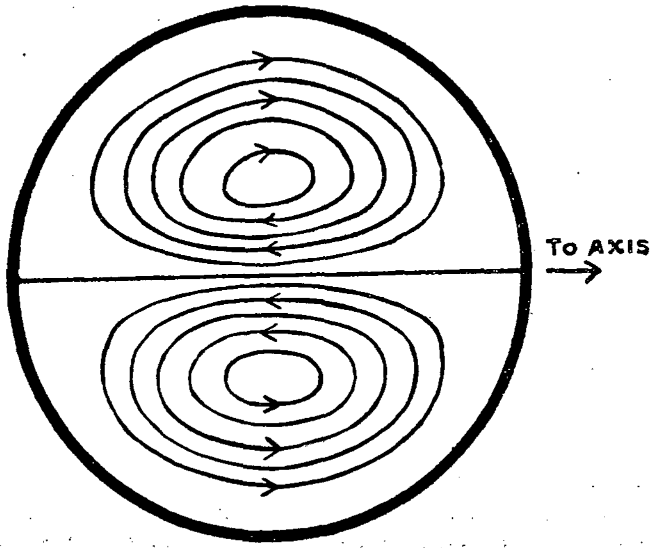

FIG A

Double Helical Circulation

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The liquid flows into two helices in the tube. These two helical streams are symmetrically dipsosed on both sides of the diameteral axis. It is the energy absorbed in these extra-viscous stresses set up in this double helical circulation that gives rise to this resistance to fluid flow. Besides, the fluid has to travel a longer path resulting in additional increase in resistance. Similar types of observations have been made by White[2], Eustice[3], Taylor[4], Keulegan and Beij[5], Adler[6] and Dryden[10].

Hawes[7] confirmed the above observations and showed that the double-circulatory effect existed up to a Reynolds number of 6,000 in case of water in a coil of curvature ratio 0.1. His observations were surprising in that earlier work had shown that velocity at a bend in a pipe is greater at the inside of the curve. By a study of models of velocity and temperature distribution in a coil he showed that the point of greatest velocity is nearer the outside wall of the curved pipe and naturally the fluid film on the outside of the curve is thinner than on the inside. These plasticine models of Hawes were a direct proof of double helical theory. These proved that turbulence in coiled pipes is of a different nature.

The increase in friction due to coiling was expressed by Dean[1] in an equation of the type:—

$$\frac{1}{C} = 1 - \left[1 - \left(\frac{11 \cdot 6}{\phi} \right)^{0.45} \right]^{2.22} \quad \text{where } \phi = \left(\frac{dv\rho}{\mu} \right) \sqrt{\frac{d}{D}}$$

for the case of stream line motion.

white $\frac{2a}{5}$ proposed the equation:—

$$\frac{F}{\rho v^2} = 0 \cdot 04 \left(\frac{dv\rho}{\mu} \right)^{-0.25} + 0 \cdot 006 \left(\frac{d}{D} \right)^{0.5}$$

For correlating data, obtained on fully coiled helical pipe. Spiers[8] proposed the relation $f' = fe^{2\pi d/D.}$ According to Perry, the data available for turbulent flow in coils of sharp curvature do not suffice to formulate a rule. White showed that true turbulence in coils may set in at much higher Reynolds number than for straight pipes. This was confirmed by Inglesent and Storrow[10], who found that in a coil with $d/D = 0.0433$, the critical Reynolds number was 7600. This value was also confirmed by measurement of energy dissipated in friction in the coil. Both isothermal and nonisothermal data agreed on this value. It was assumed therefore that double helical motion does not cause increase in the energy dissipation. Figure B gives the results on the transition Reynolds numbers obtained by various workers for coils of various curvature ratios 9A.

Although many observers have shown that transition from double-helical to turbulent flow begins at a Reynolds number of 6000, White[2] showed that true turbulence in smooth coils may not be established even up to Reynolds number of 20,000. This is because turbulence is less readily developed in coil pipes than in straight pipes. This was later confirmed by Inglesent and Storrow[10] who termed the region between the transition Reynolds number and the Reynolds number at which true turbulence sets in as semiturbulent region. Investigations by the author have confirmed the above observations for two coils with d/D of 0.0417 and 0.103.

The problem of heat transfer in helical coils under different conditions has been investigated by various authors. The earliest work reported in this connection is that of Richter[18] who found that for

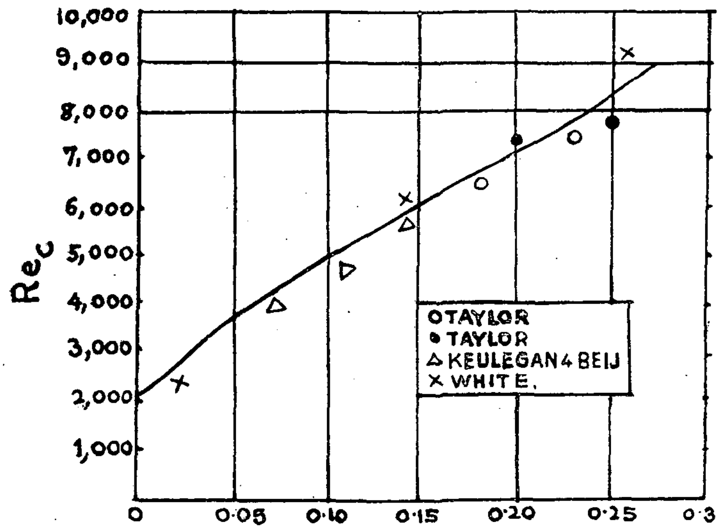

FIG. B.

$\sqrt{d/D}$

Critical Reynold's numbers for flow in coils.

a double helical water to water type heat exchanger, overall heat transfer coefficients obtained are 20% higher, than those which can be obtained for straight pipes under similar operating conditions. He however did not attempt a correlation of film coefficients of heat transfer on the basis of his data.

Jeschke[11] cooled air in two helical coils with the turbulent air flow ranging up to $\frac{dG}{\mu}$ of 150,000. Based on his data, he proposed an equation:—

$$\frac{h_i\, d}{k} = \left[0\cdot039 + 0\cdot138\left(\frac{d}{D}\right)\right]\left[\frac{dGC\rho}{k}\right]^{0.76}$$

This equation will not be valid for liquids, since the viscosity term is not included in it.

Comprehensive data for forced convection heat transfer on the outside of the coil and the inside surface of a vessel with the liquid agitated by a flat paddle were first presented by Chilton, Drew and Jebens[12]. Their correlation equation for predicting heat transfer coefficients on the outside of the coil and for forced convection is as follows:—

$$\frac{h_c\, D_j}{k} = 0\cdot87\left(\frac{L^2 \nu \rho}{\mu}\right)^{0.62}\left(\frac{C\rho\, \mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_c}\right)^{.14}$$

where $\frac{L^2{}_{v}\rho}{\mu}$ is the modified Reynolds number. This equation incorporates all the dimensionless variables with regard to the equipment in the constant 0.87 which restricts its applications to the apparatus geometrically similar to the equipment used by them. An improvement in this method of correlation was given by Pratt[13]. In the absence of complete information about the effect of curvature of a pipe on the internal film coefficient of heat transfer, he defined h_i

for coiled pipes by a more precise equation of the following type :— h_i (coil) = $\beta \times h_i$(st.pipe) where β is the curvature coefficient. The value of β obtained by him was $1+3.4$ (d/D) h_0 values were then calculated by graphical analysis. He correlated these h_0 values by the equation :—

$$\frac{h_c L_v}{k} = \alpha \left(\frac{L^2 s \rho}{\mu}\right)^{0.5} \left(\frac{d_g}{d}\right)^{0.8} \left(\frac{W}{d_c}\right)^{0.25} \left(\frac{L^2 d^v}{d_c^3}\right)$$

wherein $\alpha=39$ for square tanks and 34 for cylindrical tanks. He found that both relations hold good only for turbulent flow, for which Reynolds number must be at least 20,000.

McAdams[14] had suggested earlier that it is sufficient to multiply h_i for straight pipes by a factor ($1+3.54$ d/D). Values of h_i for coils may be therefore calculated by the following equation :—

$$\frac{h_i d}{k} = 0{\cdot}023\left(1 + 3{\cdot}54\frac{d}{D}\right)\left(\frac{dv\rho}{\mu}\right)^{0.8}\left(\frac{C\rho\mu}{k}\right)^{0.4}$$

Hawes[6] gives data for one coil of d/D= 0.1 in which, for water flowing turbulently through the coil, it is shown that the Nusselt number is directly proportional to the Reynold number. By a comparison of fluid flow and heat transfer curves, he showed that in the double helical region and increase in the film coefficient of five times over the straight pipe is obtained at the same Reynolds number, while the increase in resistance to fluid flow is only 3.5 times. His experiments were carried out a very low heat fluxes and are not therefore of general applicability.

The suggestion of McAdams was put to a more regorous test in a series of experiments on a small copper coil, by Storrow[9] and his collaborators.[10] They studied the case of heat transfer by natural convection around the coil, with water flowing through it at various velocities. In their experiments, the coil was immersed in a still hot water bath and the heat abstracted by cooling water flowing through the coil at a constant rate was measured at different intervals of temperature as the bath water cooled down slowly. In these experiments, water velocity ranging from 2 to 6 ft./sec. and Reynolds numbers from 7000 to 12000 were used. He obtained Wilson plots of $\frac{\mu^{0.4}}{v^{0.8}}$ from which h_o values were computed. It was found that these values were greater than those accepted for a sigle horizontal cylinder under the same operating conditions. He explained this fact as being due to the greater pumping action of the coil, resulting in increased conductance in lower turns of the coil although they are surrounded by water at slightly lower temperatures. He did not give a general correlation for the pediction of h_o values under a given set of conditions but observed that such a positive deviation of transfer rates in coils from those for single horizontal cylinder would be function of the number of turns and could be accounted for by introducing a function involving the length of the pipe diameter and the helical diameter.

The conclusions of Storrow that greater h_0 values are obtained for coils was later confirmed by Scott[15] who calculat- h_0 values for the coil by the equation :—

$$\frac{h_c d}{k_f} = \frac{1}{\alpha}\left(\frac{d^3 \rho^2 \beta g \Delta t}{\mu_f^2}\right)^n \left(\frac{C\rho \mu_f}{k_f}\right)^m$$

where h_c=convection coefficient of heat transfer.

k_f = thermal conductivity of fluid at film temperature.

α=emirical constant.

Δt=Temperature difference between surface and liquid.

d=shape factor.

β = coefficient of expansion of the fluid.
g = gravitational constant.
ρ = density of fluid.
$C\rho$ = sp. heat of fluid.
μ_f = absolute viscosity at film temperature.

In this equation the values used for constant α, n and m were 2.6, 0.27 and 0.25 respectively. The agreement between calculated and observed values was excellent and confirmed by subsequent experiments.

Although such a confirmation of the results of Storrow *et al.* on the rates of heat transfer by natural convection around coils was available, their quantitative aspects were considered of doubtful accuracy due to the use of the extrapolation technique based on the presumption that the correlation equation suggested by McAdams for coils applies in the range of Reynolds Nos. used by him in his tests. Further experiments in the same laboratory to decide the above matter and to assess the value of the extrapolation method confirmed this doubt. It was found that the range of Reynolds numbers covered in his experiments was insufficient to produce the fully developed turbulent region in the curved path through the coil. It was shown that for Reynolds numbers above 20,000 McAdams equation for coils holds good, but was incorrect for the range of Reynolds numbers from 7000 to 12,000. They termed this region as semiturbulent.

For the semiturbulent range of Reynolds numbers, the inside liquid film coefficients were shown by Manackerman and Storrow[10] to be given numerically by the equation :—

$$\frac{h_i d}{k} = 0{\cdot}4\left(\frac{d\nu\rho}{\mu}\right)^{0.5}\left(\frac{C\rho\mu}{k}\right)^{0.4}$$

This equation was not tested rigorously and it rested, as Inglesent has pointed out, merely on the numerical agreement with the measured values of h_i the measured effect of water velocity and thirdly on the assumption that the Prandtl number would have the same power function (0.4) as in the case of straight pipes. In addition the experiments conducted in these investigations were limited to small changes in viscosity. Direct values of h_0 obtained were then compared with those obtained by extrapolation technique. On such comparison, it was found from the value obtained by extrapolation plots of the type

$$\frac{1}{U}\ vs.\ \text{either}\ \frac{\mu^{0{\cdot}4}}{v^{0{\cdot}8}},\ \frac{1}{v^{0{\cdot}5}},\ \frac{\mu^{0{\cdot}4}}{v^{0{\cdot}5}}\ \text{and}\ \frac{\mu^{0{\cdot}1}}{v^{0{\cdot}5}};$$

that it was difficult to decide the correct abscissa function to be used in the plotting technique. For an analysis of viscosity function to know its effect on the inside film coefficient values and to test the validity of the equation, the comparison of the plots of

$$\frac{1}{U}\ vs.\ \frac{\mu^{0{\cdot}4}}{v^{0{\cdot}5}},\ \frac{1}{v^{0{\cdot}5}},\ \text{and}\ \frac{\mu^{0{\cdot}1}}{v^{0{\cdot}5}}$$

was made. As a result of such analysis Inglesent showed that for this range of Reynolds numbers the choice for the power of Prandtl number is of greater importance than that for the Reynolds number and that the correct abscissa function lies in between

$$\frac{\mu^{0{\cdot}1}}{v^{0{\cdot}5}}\ \text{and}\ \frac{\mu^{0{\cdot}4}}{v^{0{\cdot}5}}$$

and nearer the latter. Thus on the basis of such a comparison only an approximate idea of the correct function could be obtained. These data do however indicate one fact clearly that h_0 values for coils are greater than those for straight pipes and can be expressed in terms of the geometrical design of the coil.

In the investigations of Rhodes,[17] rates of heat transfer from the surface of a steam coil to surrounding water which was either still or mechanically stirred, were determined. These experiments showed that the overall heat transfer rate between steam condensing in a coil of 3/4" iron pipe and mechanically stirred water surrounding the coil is approximately 60 B.T.U./hr. °F sq. ft. The rate of heat transfer increases slightly with increase in steam temperature. When the water surrounding the coil is still, the overall heat transfer rate is from 36 to 40 B.T.U./hr. °F. sq. ft. When the water is not stirred mechanically, the heat transfer rate increases rather rapidly with an increase in steam temperature. This is according to expectations since with higher steam temperature, natural convection around the coils increases. Rhodes did not put forward any correlation equation for such heat transfer rates.

From what has been said so far it will be observed that information on fluid friction through coils is far from complete. Similarly although a great deal of work has been reported for heat exchange rates in coil type heat exchangers under diverse conditions of operation, there still remains much that can be done. Two specific points will make this evident. The effect of number of turns on the fluid friction and heat transfer have not received so far the attention that they deserve. No data are also available on heat transfer rates for the case of heat transfer from vapours condensing on the outside of the coil to fluids flowing through them. To conclude, it is hoped that this article will help to revive interest in elucidating these points and the like.

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